Elimination of Square Roots from Ballistics Equations

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Nomenclature

X = range
Z = altitude
t = time
H = drag function
g = gravitational constant
c = ballistics parameter
ρ = local air density
V = instantaneous speed
C_D = ballistics coefficient
M = Mach number
a = local speed of sound

Introduction

THIS Note contains a simple procedure which eliminates the necessity for computing square roots in the numerical solution of ballistics equations used for endo-atmospheric trajectory prediction. This results in an appreciable reduction in computer time for those machines which do not have a hard-wired square root function. The technique was developed for use in an airborne weapon delivery system where computer time was at a premium.[1,2] The procedure should prove useful in other ballistic applications.

Typical Ballistic Model

To illustrate the principle involved it is sufficient to consider a particular set of ballistics equations. Under several standard simplifying assumptions the equations of motion for an endo-atmospheric ballistic object can be written as

$$\begin{aligned} \ddot{X} + H\dot{X} &= 0 \\ \ddot{Y} + H\dot{Y} &= -g \end{aligned} \tag{1}$$

(See Ref. 3 for a derivation). The drag function H is given by

$$H = c\rho V C_D \tag{2}$$

The basic relation for speed is

$$V = (\dot{X}^2 + \dot{Y}^2)^{1/2} \tag{3}$$

The ballistics coefficient C_D is a function of Mach number and is usually given in tabular form.

In the step-by-step numerical integration of the system (1) the value of H is required several times per integration step. The computation of H requires a corresponding value for Mach number so that the appropriate value of C_D can be calculated. Mach number is given by

$$M = V/a \tag{4}$$

This means that V must be computed via a square root subroutine several times per integration step. In this context the square root function requires a relatively large amount of computer time. The need for a square root computation can be eliminated by a simple transformation of variables.

Reformulated Ballistic Model

Suppose that we multiply the denominator and numerator of Eq. (2) by a and use the definition of Mach number in Eq. (4) so that H becomes

$$H = c\rho a M C_D \tag{5}$$

Since C_D is a known function of M it is easy to obtain MC_D as a function of M^2. For instance, if C_D is given in tabular form we can readily construct a new table giving MC_D vs M^2. Once this has been done Eqs. (1) can be numerically integrated without computing any square roots since H can be computed using V^2 rather than V.

Discussion

The computation time saved using the previous transformation is, of course, highly dependent on the particular computer used and the nature of the square root subroutine. In the weapon delivery algorithm discussed in Ref. 2 the computation time was reduced approximately 15% by using the suggested transformation.

The transformation can also be used to remove square root terms from the state-vector used to estimate ballistic re-entry vehicle parameters. In particular, terms involving V can be eliminated from the transition matrix used to update the covariance equation.[4]

References

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Mass Properties of Sphere-Cone Entry Vehicles

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Introduction

IN the initial tradeoffs for an entry vehicle design it is often necessary to know the vehicle mass properties to evaluate dynamic behavior. These are not generally available and some quick method for estimating their value is desirable. Complications arise because a minimum level of static stability is usually required which dictates the mass distribution within the vehicle, while booster payload capability often places

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